Conclusion of a Probabilistic Model to Evaluate Motorcars Recycling and Dismantling Process in a Scarce Oil Environment

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Abstract **- The M|G|∞ queue system, at which customers arrive according to a Poisson process at rate λ, is considered in this study. Using this tool it is built a model to analyse a situation in which motorcars arrive at the system when getting idle, in a scarce conventional energy ambience, and leave it as soon as they are recycled or dismantled. Both situations are modelled with the same purpose. The model allows concluding that when the rhythm of dismantling and recycling of motorcars is greater than the rate at which they become idle, the system has a tendency to get balanced.**

Keywords - *Motorcars, recycling, dismantling, infinite servers queues, hazard rate function.*

1. Introduction

The humanity has come to the beginning of a new stage of development. Things have begun to change about environmental and ecological attitudes and about criteria and conditions for business, see for instance Hardin (1968), Filipe, Coelho and Ferreira (2006), Ferreira, Filipe and Chavaglia (2014). Now companies and governments seem to be concerned about the environmental changes; on this subject see Kunstler (2006). Very important changes in the way of living and in the balances of the planet are coming up fast, see Hardin (1968). Conventional sources of energy are exploited at a very fast rate and they will be exhausted in a near future, maybe within just a few decades.

The world steps far to a new era of energy. Many of the non-renewable resources are over-exploited and conventional sources of energy such as oil, gas and coal, that have been determinant as fuel's civilization in the last centuries, will collapse soon. Possibly firstly oil, then gas and lastly coal. New opportunities for business are becoming visible and began to be experienced all over the world associated to the new sources of energy.

General problems of environment have emerged from the bad use of this kind of resources; see again Filipe, Coelho and Ferreira (2006). Demand for inputs by industry companies has seriously increased for more than the two last centuries to satisfy all the demand that has resulted either from the strong increasing of human global population or from the increasing level of life for an important part of the world population. All the wastes people have made for many decades must be overcome and it should be know how to convert old equipment in useful devices, when it is possible, see Ferreira, Filipe, Coelho (2008), Ferreira *et al* (2008). Many kinds of new problems will occur and it is important to know how quickly general changes may happen, while today societies develop new sources of energy in order to create a new economy and a reorganized society.

The aim of this paper is to show that motorcars which work on the basis of oil may have an alternative use when this conventional source of energy collapses; or simply they may become dismantled. In the model to be presented, through the use of infinite servers queues, it is considered that too many motorcars will become idle if conventional energy misses or even when conventional energy becomes replaced by a renewable one. Motorcars dismantle or recycling will become very usual because there will not be a way to get them functional with conventional oil, since the moment it gets depleted.

It will be stated that it is essentially relevant the cadence at which the recycling and dismantling actions are performed, being important in this analysis the service hazard rate function, see Ross

(1983).

In this paper it is retrieved the work presented in Ferreira, Filipe, Coelho (2008), Ferreira *et al* (2011), Ferreira *et al* (2008). It is completed in the part of the model rising and enlarged in the economic analysis.

2. The Model

In the $M|G|\infty$ queue, see Takács (1962), Ferreira (1995), customers arrive according to a Poisson process at rate λ and each one receives a service which length is a positive random variable with distribution function $G(.)$ and mean value α. There are infinite servers, that is: when a customer arrives it always finds an available server. The service of a customer is independent from the other customers' services and from the arrivals process. An important parameter is the traffic intensity, called $\rho = \lambda\alpha$. The M|G|∞ queue has neither losses nor waiting. Note that there is no queue in the usual sense of the word.

In what concerns this study, the costumers are the motor cars that become idle. The arrivals rate is the rate at which the motor cars become idle. The service time for each one is the time that goes from the instant they get idle till the instant they become either recycled or dismantled.

Call $N(t)$ the number of occupied servers (or the number of customers being served) at instant t , in a M|G|∞ system. From Ferreira (1995), $p_{0n}(t) = P[N(t) = n|N(0) = 0]$, $n = 0,1,2,...$, is given by:

$$p_{0n}(t) = \frac{\left(\lambda\int_0^t [1 - G(v)]dv\right)^n}{n!} e^{-\lambda\int_0^t [1-G(v)]dv},$$

$n = 0,1,2,...$ (1).

So, the transient distribution, when the system is initially empty, is Poisson with mean $\lambda\int_0^t [1 - G(v)]dv$.

The stationary distribution is the limit distribution:

$$\lim_{t\to\infty} p_{0n}(t) = \frac{\rho^n}{n!} e^{-\rho}, \quad n = 0,1,2,... \qquad (2).$$

This queue system, as any other, has a sequence of busy periods and idle periods. A busy period begins when a customer arrives at the system finding it empty. Be $p_{1'n} = P[N(t) = n|N(0) = 1']$, $n = 0,1,2,...,$ meaning $N(0) = 1'$ that the time origin is an instant at which a customer arrives at the system, jumping the number of customers from 0 to 1. That is: a busy period begins. At $t \geq 0$ possibly, see Ferreira (1995):

-The customer that arrived at the initial instant either abandoned the queue system with probability $G(t)$, or goes on being served, with probability $1 - G(t)$;

-The other servers, that were unoccupied at the time origin, either go on unoccupied or occupied with 1,2,... customers, being the probabilities $p_{0n}(t)$, $n = 0,1,2,...$. Both subsystems, the one of the initial customer and the one of the servers initially unoccupied, are independent. So

$$p_{1'0}(t) = p_{00}(t)G(t)$$
$$p_{1'n}(t) = p_{0n}(t)G(t) + p_{0n-1}(t)(1 - G(t)), n = 1,2,... \qquad (3)$$

.

It is easy to check that

$$\lim_{t\to\infty} p_{1'n}(t) = \frac{\rho^n}{n!} e^{-\rho}, \; n = 0,1,2,... \qquad (4).$$

Calling the service time hazard rate function h*(t)*, according to [19]:

$$h(t) = \frac{g(t)}{1 - G(t)} \qquad (5)$$

where $g(.)$ is the probability density function associated to $G(.)$. The service time hazard rate function is the rate at which the services end. For the situation under study in this paper, is the rate at which the motorcars become either recycled or dismantled.

Proposition 1:

If $G(t) < 1, t > 0$ continuous and differentiable and

$$h(t) \geq \lambda, \; t > 0 \qquad (6)$$

$p_{1'0}(t)$ is non- decreasing.

Dem.:

It is enough to note, according to (5), that

$$\frac{d}{dt} p_{1'0}(t) = p_{00}(t)(1 - G(t))(h(t) - \lambda G(t)).$$

■

Obs.:

- If the rate at which the services end is greater or equal than the customers' arrivals rate, $p_{1'0}(t)$ is non- decreasing.
- For the M|M|∞ system, exponential service times, $h(t) = 1/\alpha$ and (6) is equivalent to

$$\rho \leq 1 \qquad (7).$$

Equation (6) evidences that if either the recycling or the dismantling rate is greater or equal than the rate at which the motorcars become idle, the probability that the system is empty at instant t, meaning it that there is no idle motorcars, does not decrease with t. So the system has a tendency to become balanced as far as time goes on.

Denoting $\mu(1',t)$ and $\mu(0,t)$ the distributions given by (3) and (1) mean values, respectively,

$$\mu(1',t)=\sum_{n=1}^{\infty} n p_{1n}(t)=\sum_{n=1}^{\infty} n G(t) p_{00}(t)+\sum_{n=1}^{\infty} n p_{0n-1}(t)(1-G(t))=$$

$$G(t)\mu(0,t)+(1-G(t))\sum_{j=0}^{\infty}(j+1)p_{0j}(t)=\mu(0,t)+(1-G(t)),$$

resulting

$$\mu(1',t)=1-G(t)+\lambda\int_0^t[1-G(v)]dv \qquad (8).$$

Proposition 2:

If $G(t)<1, t>0$ continuous and differentiable and

$$h(t)\leq\lambda,\ t>0 \qquad (9)$$

$\mu(1',t)$ is non- decreasing.

Dem.:

It is enough to note, according to (5), that $\frac{d}{dt}\mu(1',t)=(1-G(t))(\lambda-h(t))$. ■

Obs.:

-If the rate at which the services end is lesser or equal than the customer's arrivals rate, $\mu(1',t)$ is non-decreasing.

-For the M|M|∞ system (9) is equivalent to

$$\rho\geq 1 \qquad (10).$$

Equation (9) evidences that if either the recycling or the dismantling rate is lesser or equal than the rate at which the motorcars become idle, the mean number of motor cars in the system does not decrease with time. This means that the system has a propensity to become unbalanced as far as time goes on.

Note that for exponential service times, in both criteria considered, the condition $\rho\leq 1$ guarantees the system balance.

3. Cost-Benefit Analysis

In the former section it was seen how important was the role of *h(t)* and λ to monitor the way the system of motorcars recycling and dismantling is managed. To perform an economic analysis, based on the model presented behind, consider additionally *p* as the probability, or percentage, of the motorcars arrivals designed to the recycling being consequently *1-p* the same to the dismantling. Call $h_i(t), c_i(t)$ and $b_i(t), i=1,2$ the hazard rate function, the mean cost and the mean benefit, respectively for recycling when $i=1$ and for dismantling when $i=2$.

So the total cost per unit of time for motor cars recycling and dismantling is:

$$C(t)=pc_1(t)\lambda+(1-p)c_2(t)\lambda \qquad (11)$$

and the benefit per unit of time resulting from recycling and dismantling

$$B(t)=b_1(t)h_1(t)+b_2(t)h_2(t) \qquad (12).$$

From an economic point of view, quite extreme, it must be $B(t)>C(t)$ for any *t*. So results that it is interesting recycling if

$$b_1(t)>\max\left\{\frac{pc_1(t)\lambda+(1-p)c_2(t)\lambda-b_2(t)h_2(t)}{h_1(t)},0\right\} \qquad (13)$$

. .

If $G_1(t)$ and $G_2(t)$ are both exponential, with means α_1 and α_2, respectively, (13) becomes

$$b_1(t)>\max\left\{(pc_1(t)+(1-p)c_2(t))\rho_1-\frac{\alpha_1 b_2(t)}{\alpha_2},0\right\} \qquad (14)$$

,

with $\rho_1=\lambda\alpha_1$.

It is interesting dismantling if

$$b_2(t)>\max\left\{\frac{pc_1(t)\lambda+(1-p)c_2(t)\lambda-b_1(t)h_1(t)}{h_2(t)},0\right\} \qquad (15)$$

.

If $G_1(t)$ and $G_2(t)$ are both exponential, with means α_1 and α_2, respectively, (15) becomes

$$b_2(t) > \max\left\{(pc_1(t) + (1-p)c_2(t))\rho_2 - \frac{\alpha_2 b_1(t)}{\alpha_1}, 0\right\} \quad (16)$$

,
with $\rho_2 = \lambda\alpha_2$.

In a more global approach, more realistic and not so extreme, consider a period of time with length *T*. Then it must be $\int_0^T B(t)dt > \int_0^T (T)dt$. It is not so simple to deal analytically with this expression as in the former situation. But, considering that $b_1(t)$ and $b_2(t)$ are both constant in $[0, T]$ with values b_1 and b_2 , respectively, it is obtained

- Recycling is interesting if

$$b_1 > \max\left\{\frac{\lambda\left[pC_1^T + (1-p)C_2^T\right] - b_2 \ln\frac{1-G_2(0)}{1-G_2(T)}}{\ln\frac{1-G_1(0)}{1-G_1(T)}}, 0\right\} \qquad (17)$$

- Dismantling is interesting if

$$b_2 > \max\left\{\frac{\lambda\left[pC_1^T + (1-p)C_2^T\right] - b_2 \ln\frac{1-G_1(0)}{1-G_1(T)}}{\ln\frac{1-G_2(0)}{1-G_2(T)}}, 0\right\} \qquad (18)$$

where $C_i^T = \int_0^T c_i(t)dt, i = 1,2$.

If $G_1(t)$ and $G_2(t)$ are both exponential, being α_1 and α_2 the respective means, (17) becomes

$$b_1 > max\left\{\frac{\rho_1\left[pC_1^T + (1-p)C_2^T\right] - b_2\frac{\alpha_1 T}{\alpha_2}}{T}, 0\right\}; \rho_1 = \lambda\alpha_1 \qquad (19)$$

and (18)

$$b_2 > max\left\{\frac{\rho_2\left[pC_1^T + (1-p)C_2^T\right] - b_1\frac{\alpha_2 T}{\alpha_1}}{T}, 0\right\}; \rho_2 = \lambda\alpha_2 \qquad (20).$$

4. Conclusions

To apply this model, and get useful conclusions, it is important to check if customers' arrivals occur according to a Poisson process, see for instance Kelly (1979). In this kind of problem this is pacific, in general, due to the huge quantity of motorcars which owners certainly will look for these services. It is essential to estimate λ and *h(t)* to get conclusive particular results about the system after the available data. A correct estimate of λ will depend also on the arrivals process to be Poisson in real. And certainly the way is to decide for a λ mean estimate for a given period of time since it easy to admit that the arrivals rate will depend on time. Also it is correct to admit that with very large populations, such as the ones dealt in these situations, the estimation of *h(t)* is in general technically complicated. Then the best to do is to estimate directly *h(t)* instead of estimating first the service time distribution followed by the consequent computation of *h(t)*.For exponential service times all this is particularly easy since in this case $h(t)$ does not depend on *t*.

From Cost-Benefit analysis performed, standing in this model, it is concluded that there are minimum benefits above which, from an economic point of view, both dismantling and recycling are interesting. And the most interesting is the one for which this minimum benefit is the least. That is: in a broader perspective it is more efficient the activity that corresponds to a lower level for the minimum interesting benefit.

The model here presented contributes for a better understanding of this kind of problems. It may be applied, with eventual modifications, to study for example some other social economic and financial problems such as unemployment (Ferreira *et al*, 2011), health Ferreira (2014a,b), pensions' funds Ferreira, Andrade and Filipe, 2012) or investment projects (Ferreira *et al*, 2011). An interesting application to repair systems, concerning technical and economic features, is considered in Ferreira (2003), Ferreira (2013), Ferreira, Andrade and Filipe(2009), Ferreira *et al* (2009), Ferreira *et al* (2011).

References

[1] Kelly, F. P. (1979) *Reversibility and Stochastic Networks*. John Wiley & Sons, New York.

[2] Hardin, G. (1968). The Tragedy of the Commons. *Science*, 162, 124-148.

[3] Filipe, J. A., Coelho, M., Ferreira, M. A. M. (2006). *O Drama dos Recursos Comuns- À*

Procura de Soluções para os Ecossistemas em Perigo. Edições Sílabo, Lisboa.

[4] Kunstler, J. H. (2006). *O Fim do Petróleo- O Grande Desafio do Século XXI*. Editorial Bizâncio, Lisboa.

[5] Takács, L. (1962). *An Introduction to Queueing Theory*. Oxford University Press, New York.

[6] Ferreira, M. A. M. (1995). *Comportamento Transeunte e Período de Ocupação de Sistemas de Filas de Espera sem Espera*. PhD Thesis discussed in ISCTE, Lisboa.

[7] Ferreira, M. A. M. (2003). A two echelons repair system modelled through infinite servers queues. Proceedings of the 21st International Conference *Mathematical Methods in Economics 2003*, 75-80.

[8] Ferreira, M. A. M. (2013). Modelling and differential costs evaluation of a two echelons repair system through infinite servers nodes queuing networks. *Applied Mathematical Sciences* 7, 112, 5567-5576. DOI:10.12988/ams.2013.38478.

[9] Ferreira, M. A. M. (2014a). *The pandemic period length modelled through queue systems*. Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XVII), 43-47. Virt, Slovakia.

[10] Ferreira, M. A. M. (2014b). $M|G|\infty$ queue system in the pandemic period study. *Applied Mathematical Sciences* 8, 73, 3641-3646. DOI:10.12988/ams.2014.45328.

[11] Ferreira, M. A. M., Andrade, M., Filipe, J. A. (2009). Networks of queues with infinite servers in each node Applied to the management of a two echelons repair system. *China-USA Business Review*, 8, 8, 39-45.

[12] Ferreira, M. A. M., Andrade, M., Filipe, J. A. (2012). Studying pensions funds through an infinite servers nodes network: a theoretical problem. *Journal of Physics: Conference Series* 394, 1, Article number 012035. DOI: 10.1088/1742-6596/394/1/012035.

[13] Ferreira, M. A. M., Filipe, J. A., Chavaglia, J. (2014). Nanotechnology and processes the nano-photovoltaic panels. *Advanced Materials Research*, 837, 694-698. DOI:10.4028/www.scientific.net/AMR.837.694.

[14] Ferreira, M. A. M., Filipe, J. A., Coelho, M. (2008). A queue model for motor vehicles dismantling and recycling in a scarce energy ambience. *APLIMAT- Journal of Applied Mathematics*, 1, 1, 337-343.

[15] Ferreira, M. A. M., Filipe, J. A., Coelho, M.. (2014). Performance and differential costs analysis in a two echelons repair system with infinite servers queues. *Advanced Materials Research*, 1036, 1043-1048. DOI: 10.4028/www.scientific.net/AMR.1036.1043.

[16] Ferreira, M. A. M., Andrade, M., Filipe, J. A., Selvarasu, A. (2009). The management of a two echelons repair system using queuing networks with infinite servers queues. *Annamalai International Journal of Business and Research*, 1, 32-37.

[17] Ferreira, M. A. M., Andrade, M., Filipe, J. A., Coelho, M. (2011). Statistical queuing theory with some applications. *International Journal of Latest Trends in Finance and Economic Sciences*, 1, 4,190-195.

[18] Ferreira, M. A. M., Filipe, J. A., Coelho, M., Andrade M. (2008). *An infinite servers model for motor vehicles dismantling and recycling in a scarce energy ambience*. Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XIV), 178-186. High Tatras, Slovakia.

[19] S. Ross (1983). *Stochastic Processes*. John Wiley & Sons, New York (1983).